Project Title: Turbulence of compressible fluid with broken Galilean invariance

Project Acronym: FLOW

Applicant Name: Matej Kecer

Project summary: In this proposed work we attempt to study field-thoeretic model of fully developed turbulence, described by Navier-Stokes equation. We will consider random force with finite correlation time, the so called colored noise. This choice leads to more realistic and more general model. However, it also leads to breaking of Galilean invariance symmetry. Similar model was worked out in [1] however, authors here considered only incompressible fluids. We study such phenomena in compressible case. We approach the problem by means of field-theoretic renormalization group and our goal is to explore the model in one loop approximation.

EXCELLENCE

Present state of subject: Understanding of fully developed turbulence is a long-lasting scientific problem of statistical physics. It is believe that all of turbulence is hidden in Navier-Stokes equation (NS). Phenomenologicaly, an important concept is to be seen, and that is symmetry restoration for high Reynolds number (Re>>1). In fluid dynamics, as Re grows, spontaneous symmetry braking occurs in such systems. On the other hand, in fully developed turbulence (the mentioned Re>>1) many of these symmetries are restored in statistical sense. Many phenomenological theories appeared over the years like those of Kolmogorov [3][4][5] trying to describe turbulent flows, however, despite their great value, they appear to be inaccurate. It is, therefore, desirable to test their predictions. One of the approaches used to do that is field-theoretic renormalization group (RG). For modelling of fully developed turbulence, it is customary to use stochastic NS with random stirring force.
Most of the work up to date has been focusing on this random force to be delta-correlated in time and on study of incompressible fluid cases. Such RG analyses can be found in [6]. More recently an articles have been written on the subject [2], describing white noise in compressible case, and finally [1] presents model of colored random force, however only in incompressible fluids.
The point of studying colored noise random force models is that such a choice is, for one, more realistic, and furthermore, more general than white noise case. However number of difficulties arise. The biggest of those is model not being invariant under Galilean transformations by set-up. Relevance of such theory is, therefore, in question, as phenomenology predicts fully developed turbulence to possess such symmetry. [1] shows that their model does predict Galilean symmetry restoration for (Re>>1). Our further work will focus on compressible turbulent flows with colored noise random force.

Scientific goal(s): Our aim is to described model of fully developed turbulence in compressible fluid with broken Galilean invariance. The goals are:

A) Prove such a model leads to Galilean symmetry restoration for relevant quantities.
B) Explore, whether our results in suitable limit agree with that of [1].
C) Explore other, physically speaking, relevant cases arising from our choice of model.

Research methodology: For modeling of fully developed turbulence it is customary to start with stochastic NS equation with external stirring random force with prescribed Gaussian statistics. Later, analogous field-theoretic model is created to the stochastic problem. Such field model is created using the so-called De Dominicis-Jansen action. In such analogy, correlation functions of stochastic model agree with Green functions of field model. After analyzing field model properly one can readily use Feynman diagrammatic technique and find solutions perturbatively. The methodology is similar to one which particle physics researchers use. Later on it is necessary to do the renormalization. This will produce the desired solutions to be analyzed.

IMPACT OF RESEARCH

Enhancing the potential and future career prospects: Being a student, this is the first project I am taking part in. For that, approval of this proposal would provide me with opportunity to enhance my skills and knowledge in physics of quantum field theory, statistical field theory, and also RG analysis. It would give me chance to cooperate with experts in these fields, gain the much needed experience, attend conferences and present our results. And that way kick-start my career as aspiring theoretical physicist.

Exploitation and dissemination of results:
Problem of turbulence is one of crucial to be solved in years to come. That is why it has been labeled as Millennium problem by the Clay institute to even prove existence and uniqueness of solutions. NS equation governs hydrodynamics, that means it deals with most fundamental questions of nature: how even air or water flow. This subject has been studied for centuries, but using field-theoretic approach, the results obtained and methods used can also help researchers in other fields of statistical physics. Similar field-theoretic methods are used in solving percolation process problems, reaction-diffusion processes in chemistry, critical phenomena and more. Therefore, acquiring more theoretical knowledge about the different aspects of such theories can be of utmost importance in helping future scientific and technological development.

IMPLEMENTATION

Work plan and tasks: We start with formulating a stochastic problem of fully developed turbulence, and choose correlations of random force accordingly so as to agree with mentioned problem. The second step is formulating field-theoretic model analogous with stochastic one. Dimensional analysis of such field model has to be made and canonical dimensions of all the fields and parameters in action functional. Thus also canonical dimensions of all the greens functions of model can be computed. Further it is necessary to compute propagators of quadratic (free-field) part of the action, and also vertex (non-linear) terms, all in Fourier space. By this we have specified the elements of Feynman diagrammatic technique. Next step consist of computing all the divergent diagrams in one-loop order. This holds for both diagrams of 2 point Green functions and 3 point Green functions (so called vertex diagrams).
After which renormalization can of the model can be performed and results presented.

Risk management: We do expect possible problems, namely an additional vertex appearing in field-theoretic formulation which would considerably increase number of diverging diagrams when using Feynman diagrammatic technique. These might prove time consuming and therefore would threaten the date of completion of the project.

References:

[1] N.V. Antonov, N.M. Gulitskiy, M.M. Kostenko and A. V. Malyshev, Physical Review E, **97**, 033101, (2018).

[2] N.V. Antonov, N.M. Gulitskiy, M.M. Kostenko, and T. Lucivjansky, Phys. Rev. E **95**, 033120, (2017); EPJ Web of Conf. **125**, 05006 (2016); **137**, 10003 (2017).

[3] A.N. Kolmogorov, Dokl. Akad. Nauk SSSR **30**(4), 299 (1941);
reprinted in Proc. R. Soc. Lond. A **434**, 9 (1991).

[4] A.N. Kolmogorov, Dokl. Akad. Nauk SSSR **31**(6), 538 (1941)

[5] A.N. Kolmogorov, Dokl. Akad. Nauk SSSR **32**(1), 19 (1941);

[6] Adzhemyan, N.V. Antonov, A.N. Vasiliev, *The Field Theoretic Renormalization Group in Fully Developed Turbulence* (Gordon & Breach, London, 1999).