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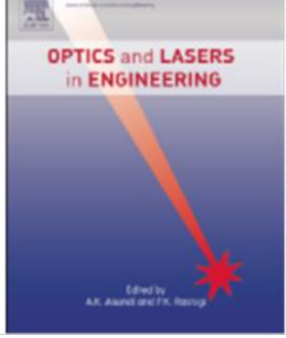
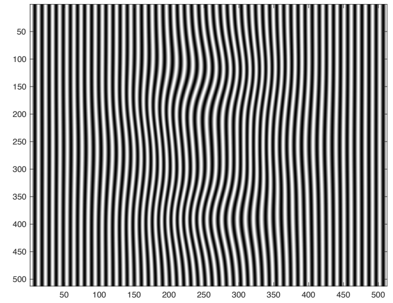


Spatial carrier fringe pattern demodulation by use of a one-dimensional continuous wavelet transform

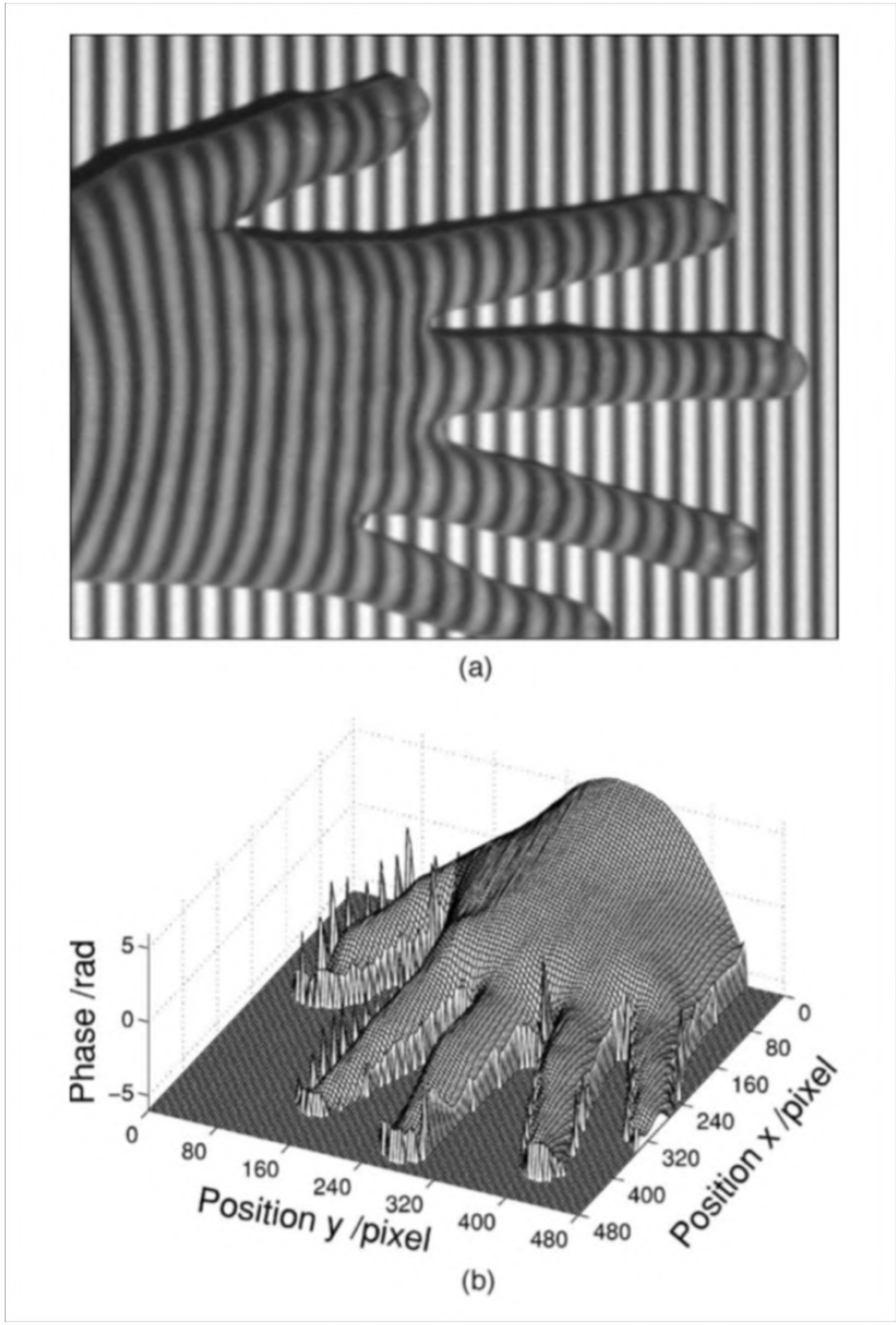
Munther A. Gdeisat, David R. Burton, and Michael J. Lalor



Jozef Haniš

Introduction

- Fringe pattern illumination
- Fourier transform:
  - Stationary processes (signals)
  - Windoved Fourier transform
  - Low resolution
- Wavelet transform (CWT):
  - Nonstationary processes (signals)
  - Estimation phase and frequencies
  - Without background illumination
  - MRA: creating new wavelet



Wavelet transform

- Mother wavelet (Morlet wavelet):

$$\psi(x) = \frac{1}{\sqrt{\pi f_b}} e^{2\pi i f_c x} e^{-\frac{x^2}{f_b}}$$

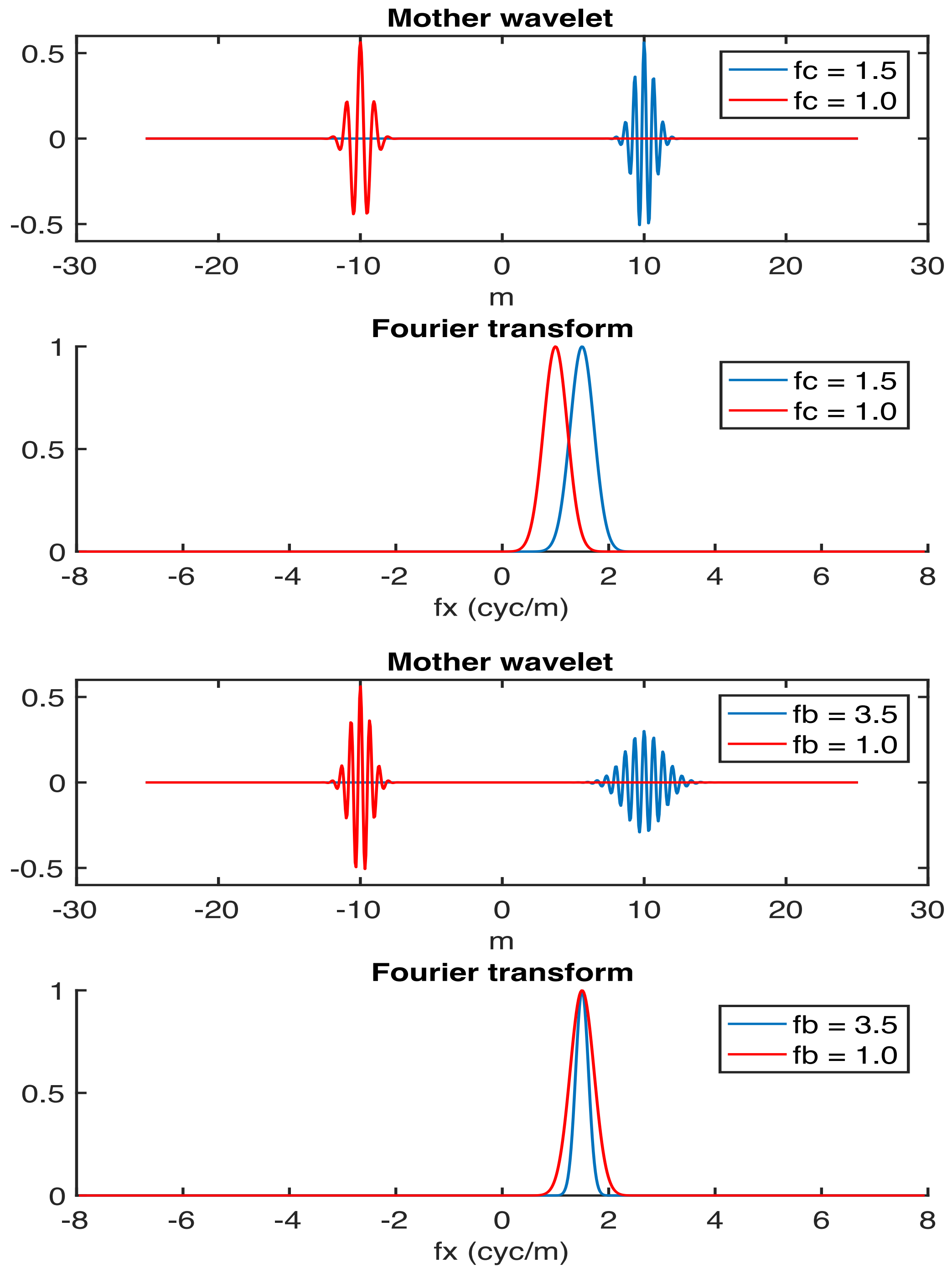
- Doughter wavelet:

$$\psi_{s,b}(x) = \frac{1}{s}\psi\left(\frac{x - b}{s}\right)$$

- Wavelet coefficients:

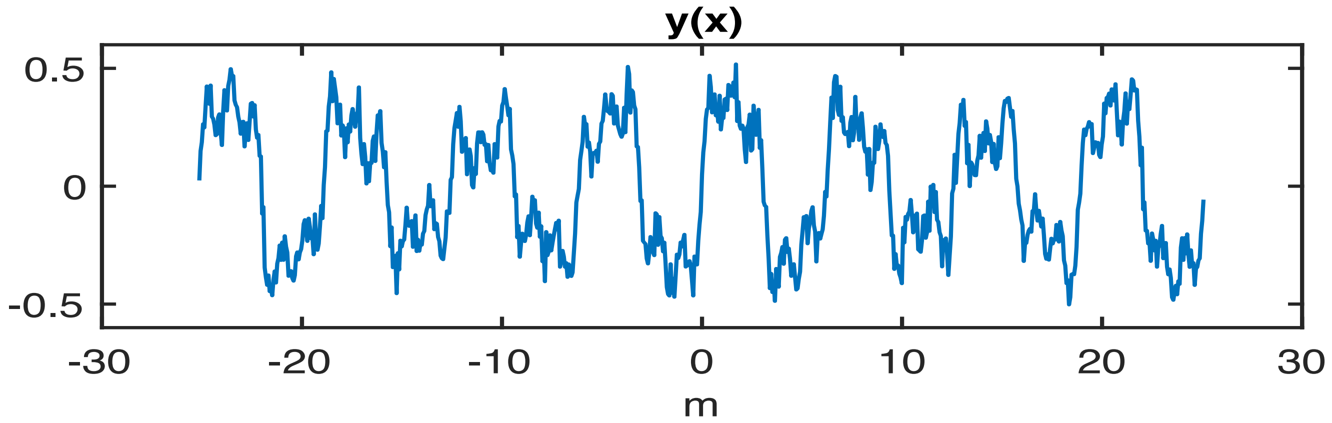
$$W(s,b) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} \psi^*\left(\frac{x - b}{s}\right) f(x)dx$$

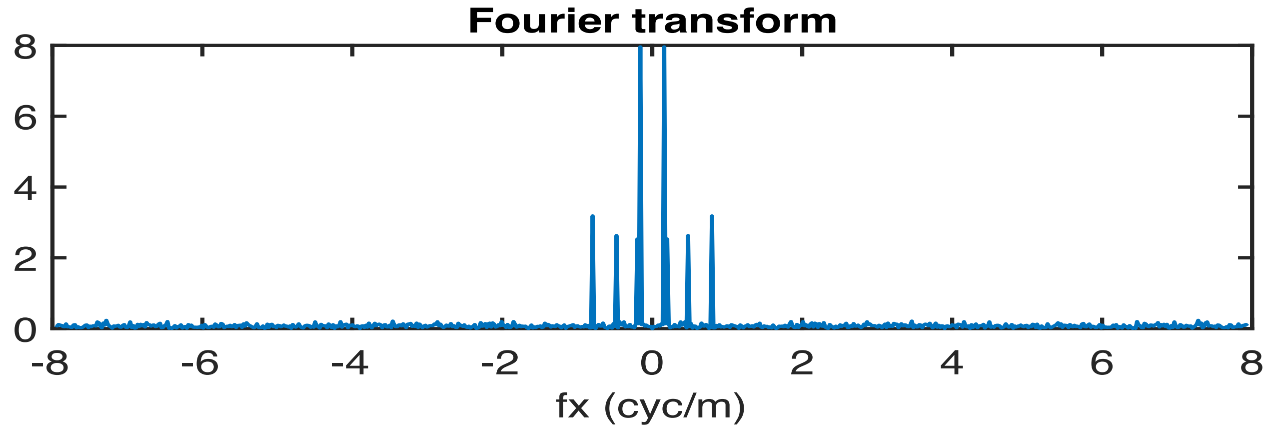
$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} \Psi^*(s\omega)F(\omega)e^{-i\omega t}d\omega$$

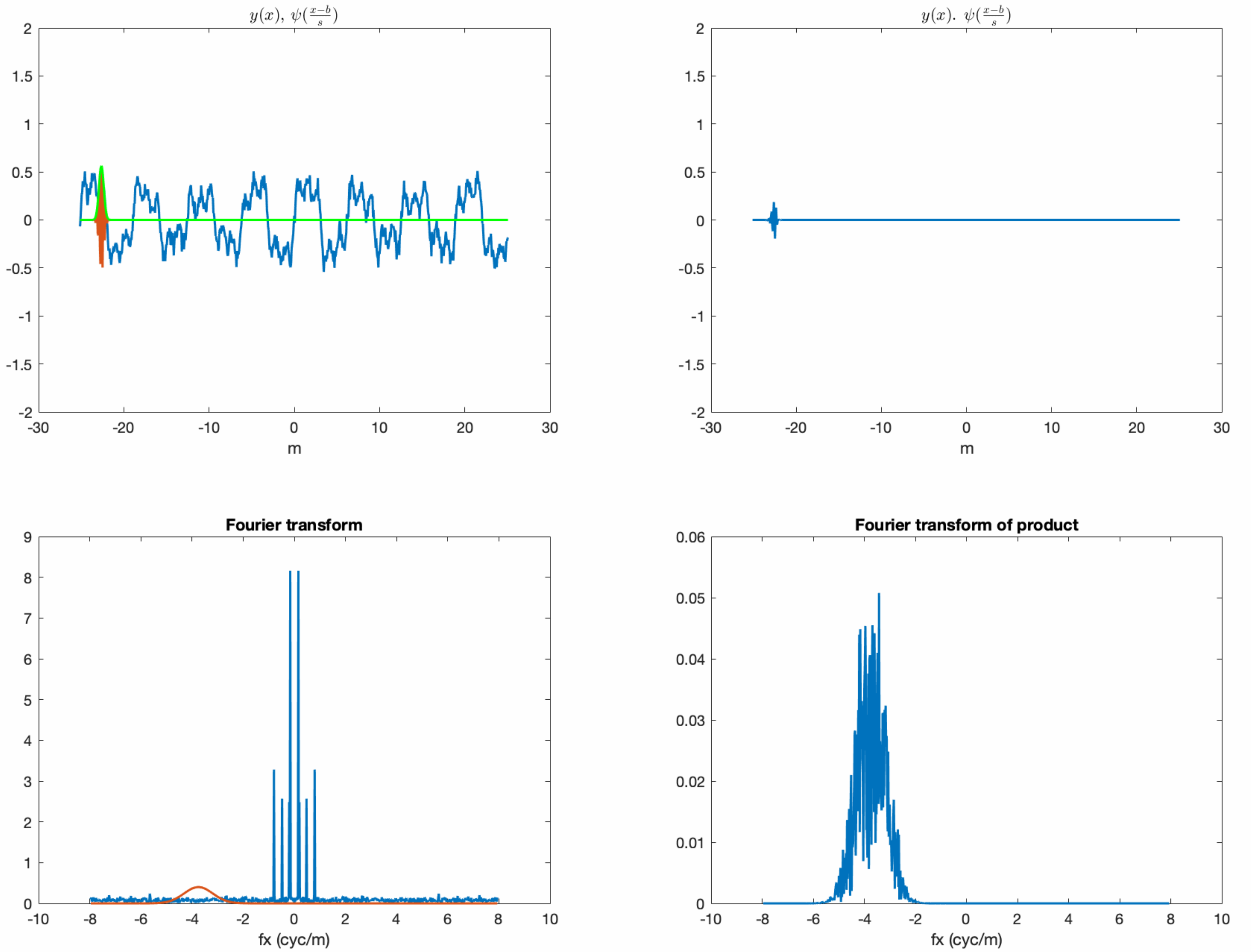


Wavelet example:

- $y(x) = \frac{1}{\pi}\sin(x) + \frac{1}{3\pi}\sin(3x) + \frac{1}{5\pi}\sin(5\mathrm{x}) + 0.1\sin\left(\frac{2\pi}{5}x\right) + \text{noise}$






$y(x),\ \psi(\frac{x-b}{s})$
m
$y(x).\ \psi(\frac{x-b}{s})$
m
Fourier transform
fx (cyc/m)
Fourier transform of product
fx (cyc/m)

Phase estimation:

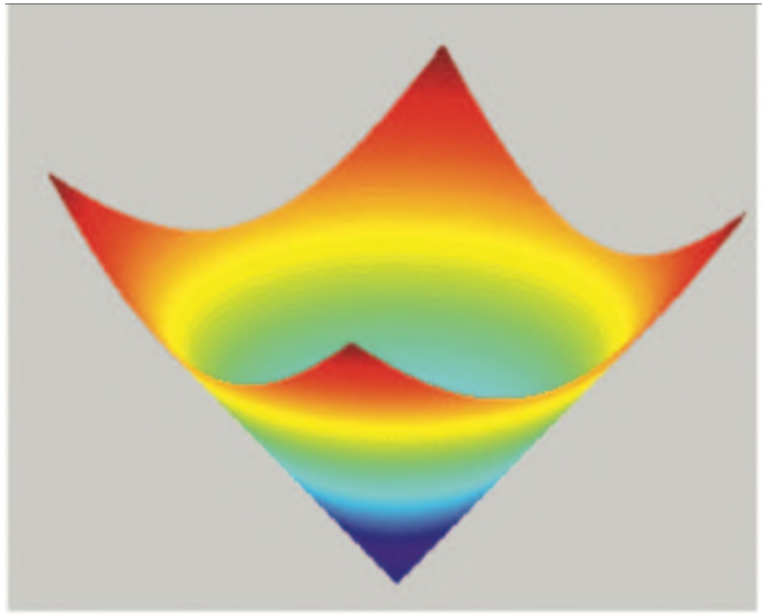
- Consider a general function: $\phi(x, y)$,
- $g(x, y) = a(x, y) + b(x, y)\cos(2\pi f_0 x + \phi(x, y))$

$a(x, y)$: background illumination

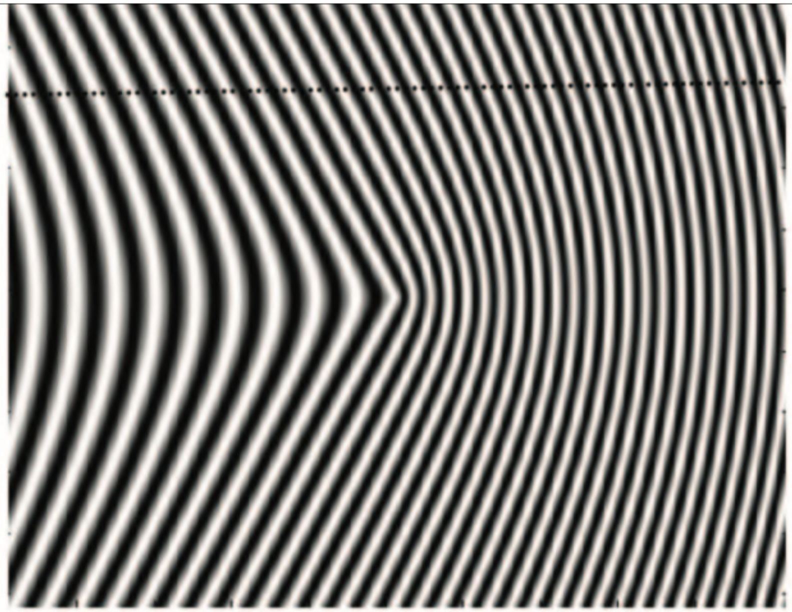
$b(x, y)$: amplitude modulation of the fringes

f_0 : spatial carrier frequency

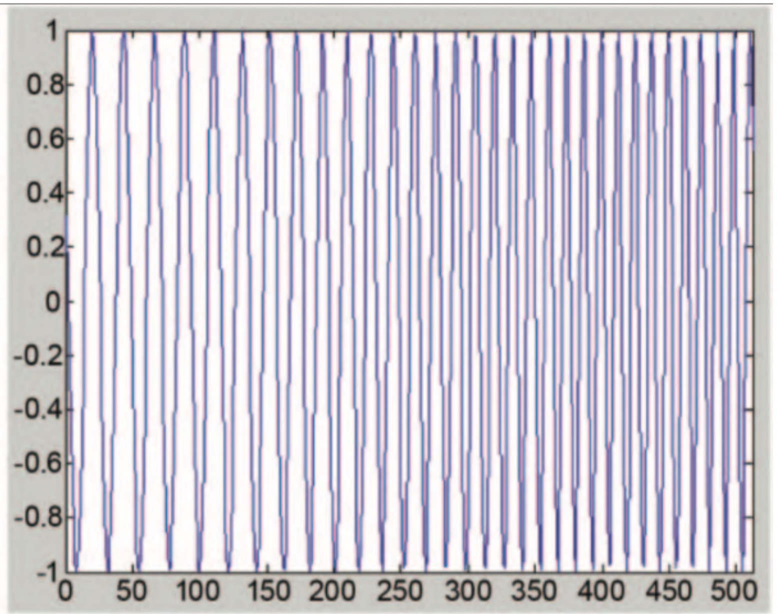
(a): computer generated object



(b): deformed fringe pattern



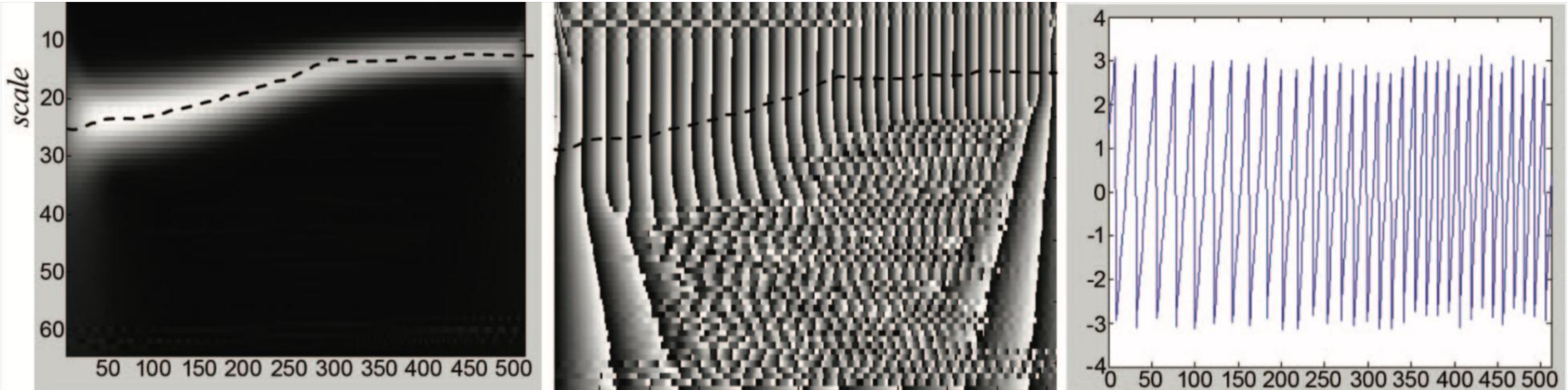
(c): intensity of fringe pattern



Phase estimation

- Maximum ridge detection
- $\varphi(s,b) = \tan^{-1}\left[\frac{\Im\{W(s,b)\}}{\Re\{W(s,b)\}}\right]$: phase of wavelet transform

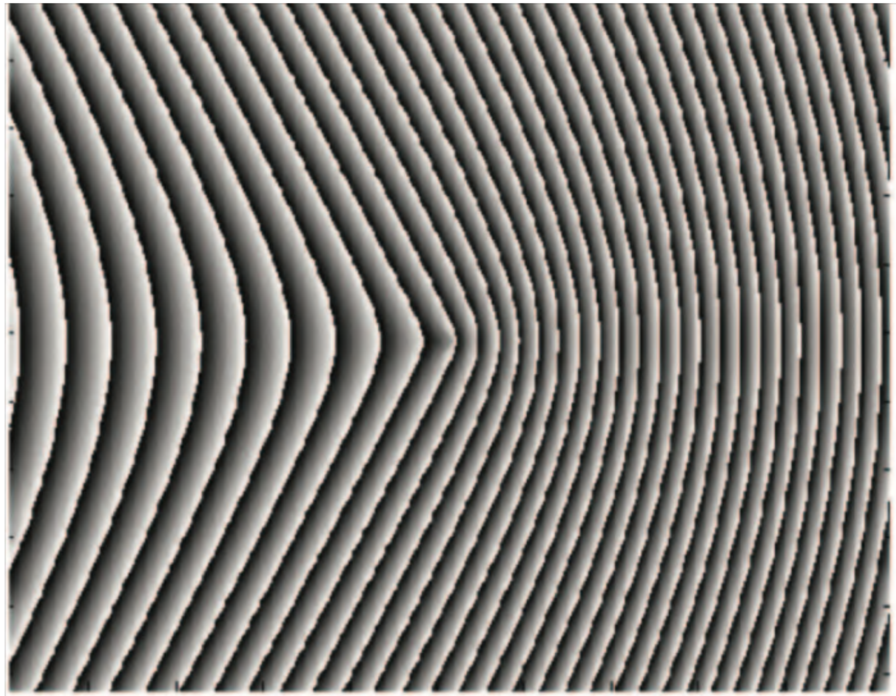
(d): modulus of wavelet coefficients (e): phase of wavelet transform (f): wrapped phase



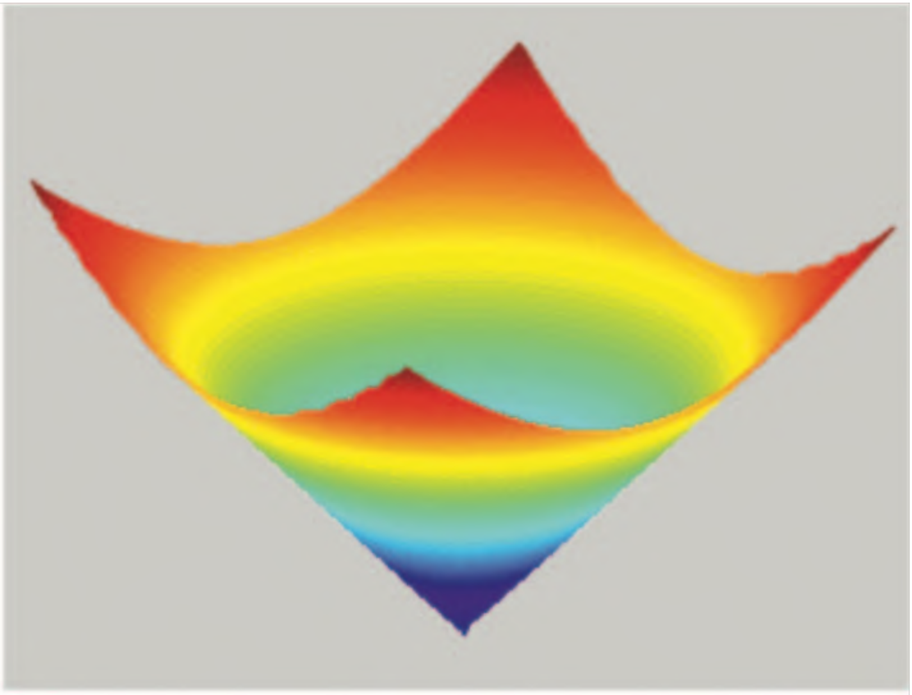
Phase estimation

- Itoh's unwrapping method

(g): wrapped phase map



(h): unwrapped phase



Another mother wavelets:

Morlet wavelet:

$$\psi_{morl}(x) = \frac{1}{\sqrt{\pi f_b}} e^{2\pi i f_c x} e^{-\frac{x^2}{f_b}}$$

Paul wavelet:

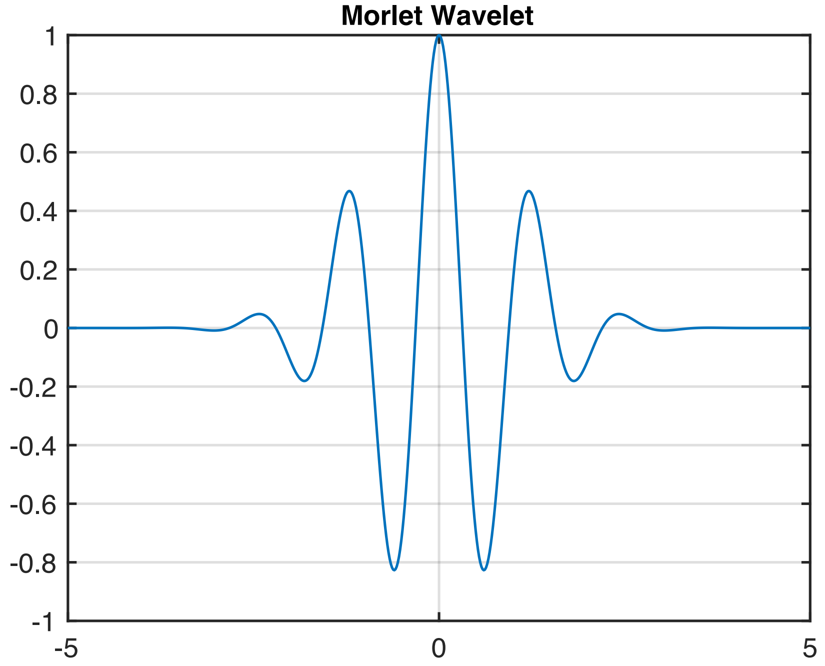
$$\psi_{paul}(x) = \frac{2^n n!(1-ix)^{-(n+1)}}{2\pi\sqrt{(2n)!/2}}$$

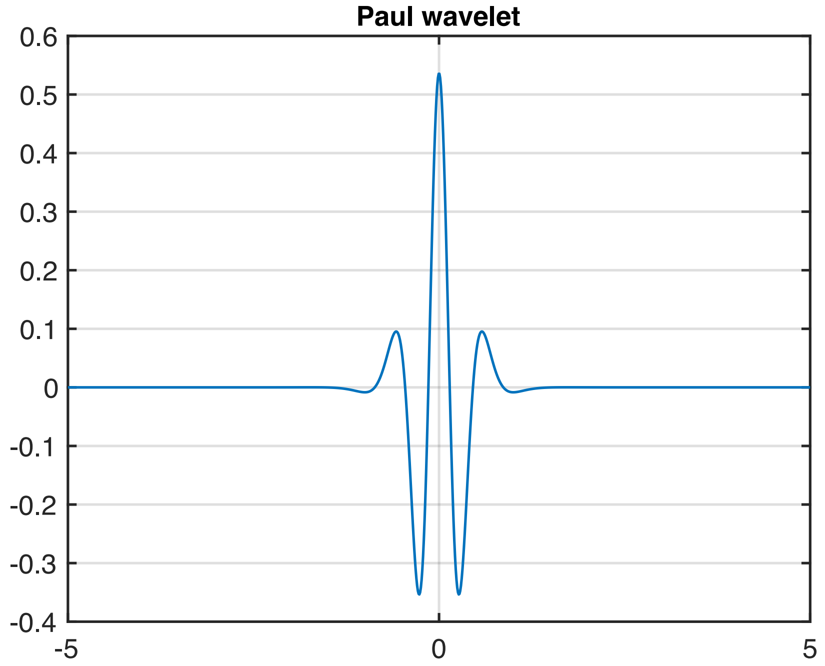
Gaussian wavelet:

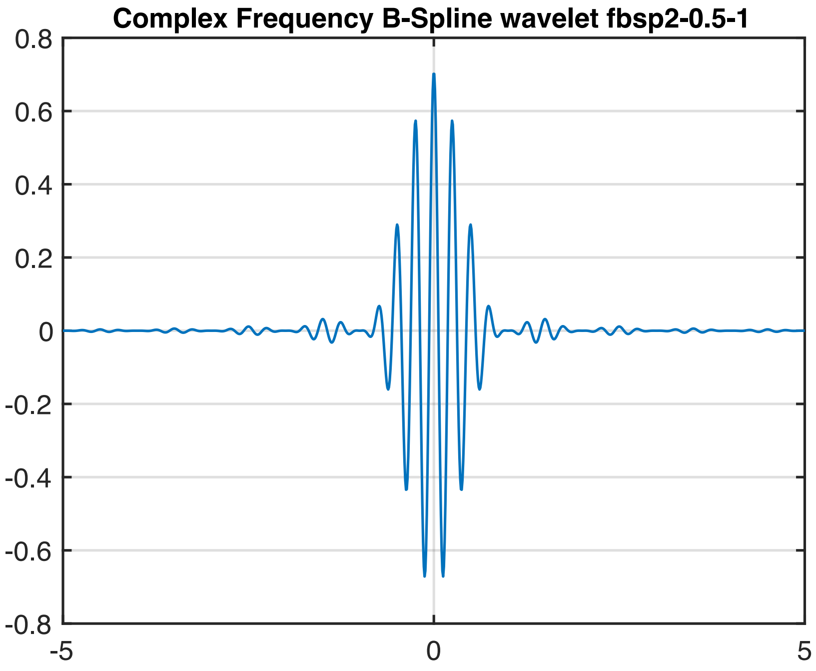
$$\psi_{gauss}(x) = \frac{d}{dx^p}\left(C_p e^{-ix - x^2}\right)^p$$

b-spline wavelet:

$$\psi_{b-spl} = \sqrt{f_b} e^{-i2\pi f_c x}\left[\text{sinc}\left(\frac{f_b x}{m}\right)\right]^m$$

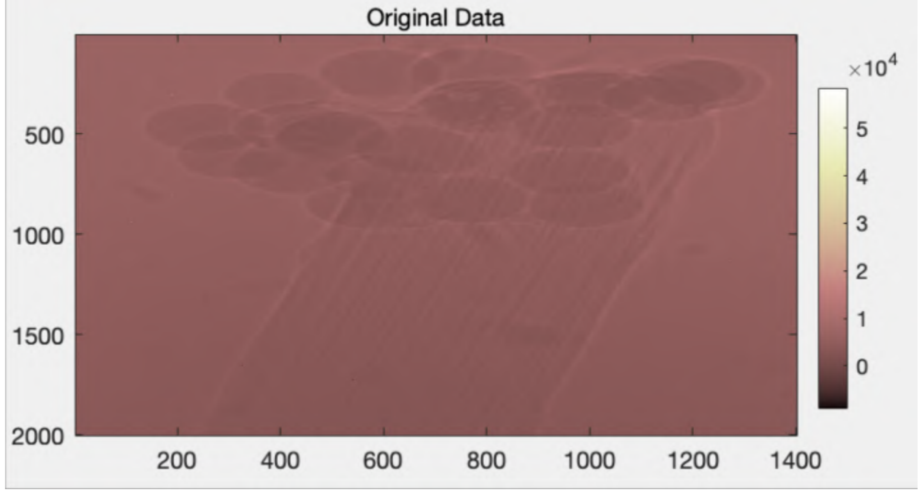




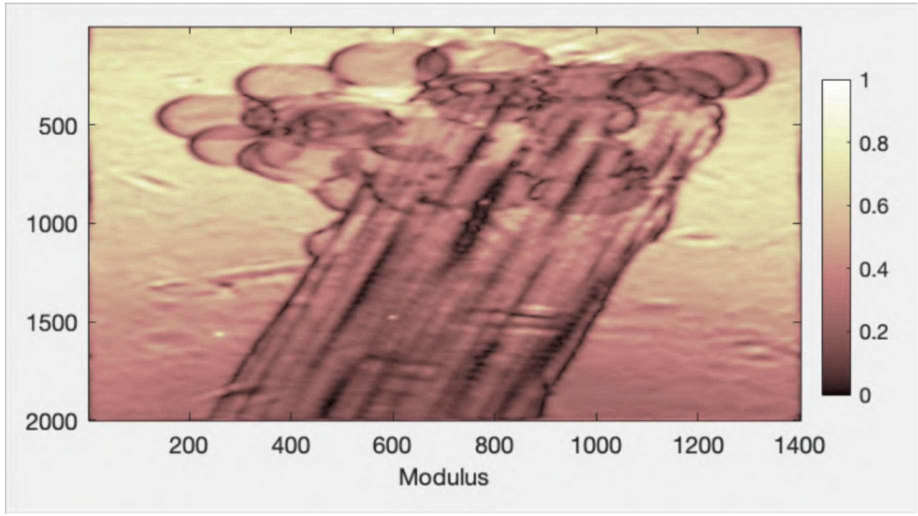

Complex Frequency B-Spline wavelet fbsp2-0.5-1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-5
0
5


Order 8 Gaussian Wavelet
1.5
1
0.5
0
-0.5
-1
-5
0
5

Conclusion



- Spatial carrier fringe pattern demodulation
- Comparison: Fourier transform & Wavelet transform
- Phase estimation:
  - Maximum ridge detection
  - Itoh‘s unwrapping method
- Advantages of Mother wavelets:
  - Morlet wavelet
  - Paul wavelet
  - Gaussian wavelet
  - b-spline wavelet

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